Lecture notes 4: The Sun as a Star i

Radiative ?ux, e ective temperature, and luminosity

Measurements of the amount of solar radiation per unit area and time that reaches the earth?s orbit give the solar constant

$$f_E = 1.36 ?\ 10^3\ \mathrm{W/m^2}, \tag{1}$$

since the area of a sphere with radius $r_E = 1$ AE=150 ? 10^6 km is $4\pi r^2$ and since we may assume that the Sun?s emission is independent of direction the total solar radiative emission, the **luminosity** is

$$L_S = f_E 4\pi r_E^2 = 3.9 ?\ 10^2 6\ \mathrm{W}. \tag{2}$$

The solar radius is easily measured once one knows the earth?sun distance; the angular solar radius is measurable to 16?, trigonometry then gives

$$R_S = r_E \tan\theta \approx r_E \theta = 698\,132\ \mathrm{km}. \tag{3}$$

We know that a body in thermodynamic equilibrium (TE) will radiate an energy ?ux $f = \sigma T^4$ from its surface. This, combined with the values of L_S and R_S derived above allow us to assign the Sun an **e ective temperature**

$$T_{\mathrm{e}} = \left[L_S/\sigma 4\pi R_S^2\right]^{1/4} = 5800. \tag{4}$$

Notice then that since the energy emitted by a star ? the luminosity! ? is conserved, knowledge of two of the three parameters, stellar radius, luminosity and stellar e ective temperature is enough to derive the third. Though it must be admitted that in order to measure the luminosity a good idea of the distance to the star is required. Why?

The temperature of the Earth

The total luminous power P intercepted by the Earth is

$$P = \pi R_E^2 (L_S/4\pi r_E^2), \tag{5}$$

where piR_E^2 is the cross sectional area of the Earth, L_S is the luminosity of the Sun and r_E is the Earth ? Sun distance. The rotation of the Earth may be accounted for spreading P over the entire surface area of the Earth $4\pi R_E^2$. Ignore the role of the atmosphere and assume that only 71% of the Suns energy is absorbed by the Earth, the rest being re?ected by cloudtops, the ground, and the oceans. The remaining energy heats the Earth and the Earth reradiates this energy as a black body at the rate σT^4 per unit area.

In equilibrium, energy input must equal energy output. This implies that the temperature can be found by equating $0.71 ?P/4\pi R_E^2 = \sigma T^4$ giving a temperature for the Earth of 255 K which is due to the fact that we have not accounted for the ?greenhouse e ect?, which keeps the average surface temperature of the Earth above the freezing point of water 273 K.

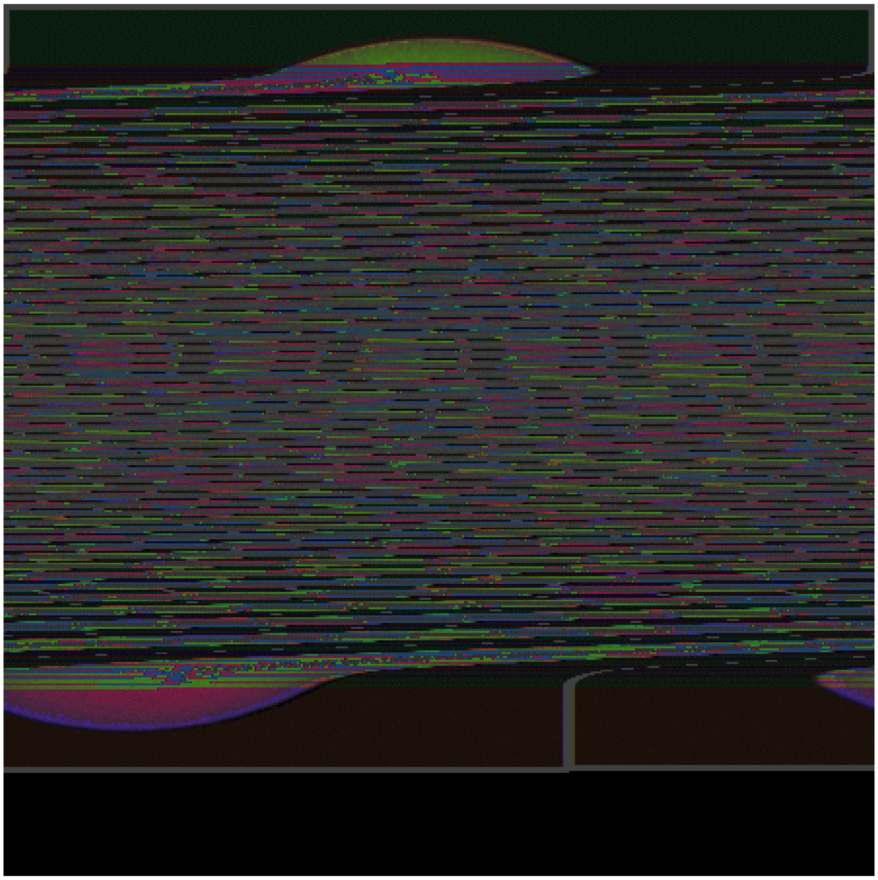

Figure 1: Limb darkening in the Solar photosphere

The Solar photosphere

A spectrum arises when one measures the run of radiated intensity with wavelength λ or frequency ν. We may check our estimate of the e ective temperature for the Sun by comparing the **solar spectrum** with the spectrum one expects from a body in TE at the same $T_{\mathrm{e}\Box}$ = 5800 K. There are detectable di erences, even in the optical region (between 400 nm and 700 nm) and there are spectral **absorbtion lines**. This is mainly due to the fact that the Sun is *not* in TE!

In fact $T_{\mathrm{e}\Box}$ = 5800 K is only typical of a small part of the solar atmosphere, the **photosphere**. This is not surprising since energy is ?owing out of the Sun, which based on our discussion of thermodynamics (the 0^{th} law) implies that the Sun must be hotter inside. Consider for example ?gure 1 which shows the phenomenenon of limb darkening.

The light we see has traversed a certain distance $\Delta\tau$ of the solar atmosphere; one sees further into the sun when viewing the center than when viewing the limb. If the temperature gradient dt/dr falls with height in the photosphere we therefore expect the observed e ect of limb darkening since high temperature gas emits more vigorously than low temperature gas. Should we expect that the *color* of the light is di erent at the limb and at sun center?

The existense of absorption lines is a consequence of the same phenomenon: and is due two physical e ects

1. A continuum of photons with various wavelengths ?ow from high temperatures at greater depths. Atoms at higher layers absorb some of these photons and go into an excited state. Deexciteation may occur collisionally; the photon energy goes into the kinetic energy of the colliding electron and the de-excited atom. Since the temperature of the gas is lower in higher layers there are statistically more such radiative excitations followed by collisional deexcitations than the opposite process of collisional excitations followed by spontanious radiative deexcitations. The net e ect is the removal of photons from the continuum radiative ?eld. This process is called **true absorbtion**.

2. The radiatively excited atom may also deexcite radiatively, with the emitted photon travelling in a di erent direction than originally. Since the Sun has a surface and we are situated outside this surface, more photons are removed from our line of sight than are emitted into it and a dark line in the spectrum results. This process is called **scattering**.

The Solar Mass

Except observations of neutrino emission and solar oscillations the only way we may gather information on the interior of the sun is through physical reasoning. This requires a knowledge of the solar mass.

Assuming that the Earth orbits the Sun in a circular orbit and that the mass of the Sun is much greater than the mass of the Earth we can set up the following from Newton?s second law F = ma:

$$\frac{GM_S m_E}{r_E^2} = \frac{m_E v^2}{r_E} \tag{6}$$

The velocity of the Earth is $v = 2\pi r_E/P$ = 29.8 km/s using P = 1 yr. This gives a solar mass of M_S = 2.0 ? 10^{30} kg, which is 3 ? 10^5 ? m_E. Dividing M_S by the solar volume gives an averege density ρ_S = 1.4 ? 10^3 kg/m^3 which is quite a bit less than the Earth?s density ρ_E = 5.5 ? 10^3kg/m^3.

Could the Sun be built up by tightly packed hydgrogen (H) atoms? If we assume that the electrons are in the lowest state such that the volume of the atom is $(2a_0)^3$, *i.e.* of order a Bohr diameter cubed, we would expect a density $\rho = (m_p + m_e)/(2a_0)^3$ = 1.4 ? 10^3 kg/m^3, which is very close the the average solar value.

Could such a gas of neutral H-atoms survive? Let us compare the force per unit area that hold these atoms together with the pressure expected in the solar core. The former is the electric force divided by the area characterizing the H-atom

$$\frac{F_e}{(2a_0)^2} = \frac{1}{4\pi\ _0}\frac{e^2}{a_0^2} ? \frac{1}{2a_0^2} = 7.6 ? 10^{12}\ \mathrm{N/m^2} \tag{7}$$

The latter may be calculated by knowledge of the solar mass and of the gravitational forces to be $P_c = 2.1 ? 10^{16}$ N/m^2 (as shown later). The pressure of the gas at Sun center are therefore much to great to allow H-atoms (or indeed any atoms) to be in a neutral state. On the other hand the distances between particles, protons and electrons, is on the order of a Bohr radius ($\approx 10^{-10}$ m) (it is actually a little less, but no matter) which is much larger than the size of protons ($\approx 10^{-15}$ m). Interparticle forces are therfore unimportant and the solar core is a perfect gas or more properly plasma since the particles constituting the gas are charged.