Lecture notes 10: Stellar evolution i

Stars, meaning globes of gas that experience Hydrogen fusion in their cores during their lifetimes, come in a wide range of masses $0.1M_S < M \; < 100M_S$.

The central temperature and density of gas globes lighter than $0.1M_S$ are too small to allow the fusion of Hydrogen, even though extremely large planets and brown dwarfs may experience short periods of Deuterium burning during their early history.

Stars heavier than roughly $100M_S$ cannot be formed since the radiation pressure from such objects is great enough to prevent the further accumulation of matter thus choking off the gas that could be used in forming heavier stars. It is now thought that some of the earliest stars, formed near the centre of proto-galaxies, early in the Universe?s history had masses on the order of $300M_S$. These stars played an important role in the reionization of the Universe, and were perhaps the seeds of the supermassive black holes we now find in the center of nearly every galaxy.

The chemical composition of stars is found from studies of photosperic spectra. Since stars, in general, do not mix their own material from core to photosphere during their lifetime (on the main sequence at least), it is thought that the photospheric abundance gives a good description of the chemical state of the matter that formed the star - *i.e.* that it is primordial. One finds that stars have Hydrogen abundances of roughly 70% (by weight, *not* by number), Helium abundances of roughly 28?30%, while the remainder consists of a mix of heavy elements. These elements, by astronomers called **metals**, can vary in abundance from 2-3% **population i stars** to 0.1 ? 0.01% **population ii stars** and perhaps all the way down to $10^{?3}$% or below in, perhaps hypothetical, **population iii stars**.

The large overabundance of Hydrogen and Helium is an indication that the Universe has had a hot past since thermodynamic equilibrium at today?s temperature would predict that nuclei would seek out bound states; *at low temperatures, material systems prefer more binding energy*.

The Hertsprung-Russel Diagram

Stars are well organized by plotting them in the so-called **Hertzsprung-Russel diagram**. In this diagram the stars luminosity L or absolute magnitude is plotted against the stars effective temperature T_{eff} or spectral class. The former ranges roughly from stars with a luminosity $10^{?2}L_S$ to stars with a luminosity 10^5L_S The latter is indicated by a letter/arabic number combination with the letters running OBAFGKM from hottest (40 000 K) to coldest (2 500 K) and each letter subdivided into ten catagories 0 ?? 9. When doing so it is found that most stars can be placed in one of five **luminosity classes**, as indicated on the right hand plot of figure 1, these classes are given a roman numeral i ? v. Thus, for example, the Sun is classified as a G2 v star.

Hydrogen burning stars are all placed on the **main-sequence**, luminosity class v, and it is the stellar mass that is the most important parameter in decid-

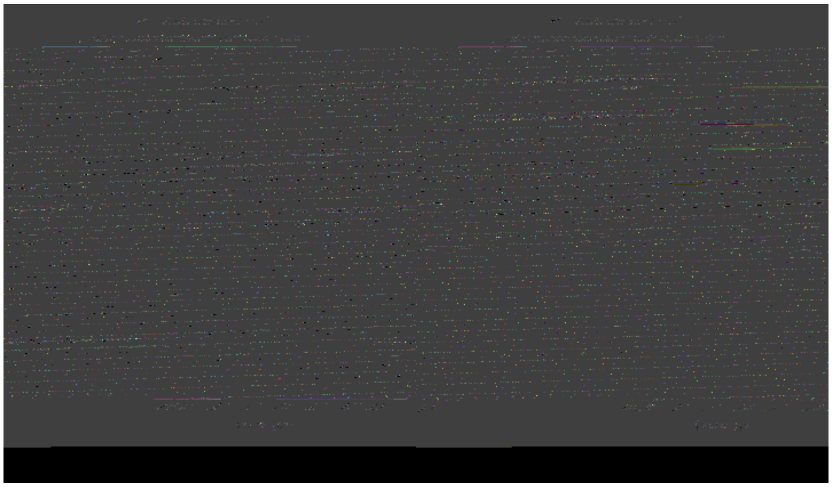

Figure 1: The **Hertzsprung-Russel diagram** with various well known stars plotted in. To the right the location of the various **stellar luminosity classes** in the HR-diagram. Source *Universe* fifth edition, by Kaufmann & Freedman.

ing where on the main sequence a star resides. (A stars chemical composition will only move a star a small bit on the same diagram).

Note that if we consider the relation between the luminosity, the effective temperature and stellar radii,

$$L = 4\pi R \ \sigma T_{eff}^4, \tag{1}$$

we find that stars of a given stellar radius lie on straight lines in the Hertzsprung-Russel diagram as shown in figure 2. It should be clear that stars with large radii are found up and to the rigth in the HR-diagram, while stars with small radii are found down and to the left. Note also that stars on the main-sequence do not vary *too* much in radius, but do become larger with increasing effective temperature and luminosity.

The Mass-Luminosity Relationship

Let us now try to derive why stars are placed as they are in the HR-diagram based on their mass, *i.e.* let us try to derive a relation between the mass and luminosity of stars. Assuming that energy is carried by radiation (and not convection) we have already derived the relation

$$L = \frac{(4\pi R^3/3)(aT^4)}{(3R^2/lc)} \tag{2}$$

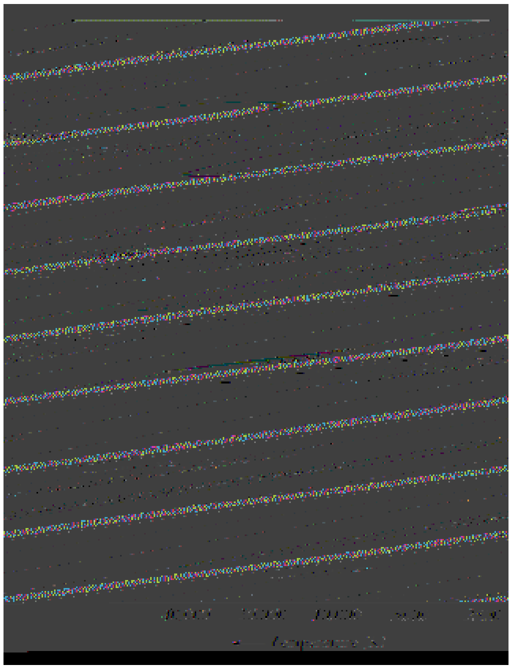

Figure 2: Lines of constant radius in the Hertzsprung-Russel diagram, derived by setting R = constant in the logarithm of the relation $L = 4\pi R^2 \sigma T^4$. Source *Universe* fifth edition, by Kaufmann & Freedman.

Let us ignore all constants and write our relation as

$$L \quad RT^4 l \tag{3}$$

.

As shown earlier, the mean free path for photons can be written as $l \quad T^{3.5}/\rho^2$ for Hydrogen *free-free* opacity and as $l \quad 1/\rho$ for electron scattering opacity. The former dominates stars of low to medium mass, the latter for stars of high and very high mass.

Furthermore, we have that the density can be written $\rho \quad M/R^3$. We also know that the pressure follows from considerations of hydrostatic equilibrium $P \quad M^2/R^4$ and can either be due the pressure in an ideal gas $P_g \quad \rho T$ or for very high mass and temperature stars the photon pressure $P_{photon} \quad T^4$.

Combining these relations gives us the following:

- Low to medium mass stars we find $T \quad M/R$, $l \quad M^{1.5}/R^{2.5}$, which give to

$$L \quad M^{5.5}/R^{0.5} \tag{4}$$

.

? High mass stars we find $T \quad M/R$, $l \quad R^3/M$, which give

$$L \quad M^3 \tag{5}$$

? Very high mass stars we find $T^4 \quad P \quad M^2/R^4$, $l \quad R^3/M$, which give

$$L \quad M \tag{6}$$

Lifetimes of Stars

Noting that Hydrogen fusion requires temperatures in the cores of stars to be on the order $15 ? 10^6$ K without varying too much, so that we can write $M/R \approx$ constant, we find that a good *average* relation for most stars, except the most massive is

$$L \quad M^4. \tag{7}$$

But the total energy content available for a star that is converting its mass to energy (via mc^2) is on the order $E \quad M$. Since the luminosity times the lifetime must equal the energy content we see that we can write

$$Lt_{life} \quad E \qquad \text{or} \qquad t_{life} \quad E/L \quad 1/M^3 \tag{8}$$

Heavy stars live much shorter lives than lighter stars.

Radii and effective temperatures

Since T_c in Hydrogen burning stars is roughly constant we have, as indicated above, $M \quad R$. Actually, for stars more massive than the Sun, the CNO-cycle dominates and since this cycle is even more temperature sensitive than the pp-cycle one finds $R \quad M^{0.6}$. In either case this implies that stars should have greater radii as we move up along the main sequence, as is observed.

Finally, using the same relation, we find that since $L \quad R^2 T_{eff}^4$ and $L \quad M^4$ that

$$T_{eff} \quad M^{\frac{1}{2}}. \tag{9}$$

Thus a star $10M_S$ will have an effective temperature $T_{eff} \approx 40\,000$ K which is blue, a star $0.5M_S$ will have $T_{eff} \approx 4\,000$ K which is red.

Stars of a given mass have their smallest sizes when on the main sequence; hence these stars are often referred to as dwarfs. (Note that this is not true when the stars become white dwarfs, neutron stars or black holes.)